Credit and Business Cycles:
Colombian Bank Competition & Credit Quantity vs Quality

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Abstract

A projection method employing finite elements and a parameterized expectations algorithm is proposed for the global approximation of the equilibrium of a dynamic stochastic general equilibrium model capable of replicating the credit cycle properties of an economy where the banking sector is constrained by a usury rate; as is the case of Colombia. The algorithm is shown to be accurate and efficient approximating highly nonlinear regions of the policy functions, specifically along the space of state variables where the slackness multiplier of the occassionally binding constraint alternates between zero and strictly positive values. The research aims at evaluating the optimal countercyclical public policies available to central governments, and deriving inferences on the Colombian informal banking sector.

JEL Classification: C63; C68; O42; E32

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1 Introduction

The imposition of an interest rate ceiling on loans originated by a non-centralized banking system, or "usury rate", is a regulatory practice exercised by selected countries with, either, a developed or emerging economy. Being Colombia an example of the latter. Economic theory recognizes, and empirical evidence confirms, that while interest rate ceilings are successful in avoiding the possibility of a specific type of usury in the formal banking sector of the economy, these policies have the potential to create significant inefficiencies in the entire financial system. These inefficiencies manifest in the volume of credits not granted to potential investors, which would have instead been granted if the financial system was not restricted by an upper limit on interest rates. As a direct consequence that these credits are not granted at the formal banking sector, national aggregate private investment is endogenous capped and an economic environment prone to the emergence of an unregulated parallel-

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banking sector is created. This parallel informal banking sector is able to grant credit opportunities to investors not served by the formal sector, at higher interest rates than that limited by the usury rate.[1]

Until recently, with the seminal work of Kiyotaki & Moore (1997), the scientific literature on DSGE models proved inefficient integrating the credit sector to the analysis of economic cycles. Kiyotaki & Moore (1997) proposes a rigorous modeling setup that considers a population of heterogeneous agents in the economic system and a credit constraint based on the collateral of the debtor. As shown in Iacoviello (2005) and Devereux & Yetman (2010), the inclusion of these innovations in a DSGE model can generate cyclical elements in the economy and produces the financial accelerator effect that the literature recognizes that is present in an economic system.

Recognizing the predicted dynamics of a model economy that is able to reproduce the amplitude and non-symmetrical components of credit cycles caused by an ad-hoc usury rate is fundamental for the development of optimal counter-cyclical monetary and fiscal policies, and for the DSGE scientific literature in general. When the proposed economic model is calibrated according to the Colombian aggregate economy, in addition to depicting the credit cycles of the formal sector of the Colombian economy, the model provides an estimate on the dynamics of the Colombian informal parallel-banking sector.

This paper constructs a DSGE model capable of replicating the amplitude, asymmetry and propagation capabilities of credit cycles in economies exogenously restricted by an interest rate ceiling. Doing this, the original model of Kiyotaki & Moore (1997) is amended to consider a more general utility function of the representative agent, a production function which, in addition to fixed capital, considers labor as a factor of production, and the imposition of an ad-hoc usury rate. The ad-hoc usury rate is built into the system of equations representing the economic system through a Lagrangian multiplier that discriminately alternates its value between strictly positive numbers and zero, depending on the state of nature of the economic environment. The multiplier will assume positive values at scenarios where the equilibrium of the economic system yields an interest rate on loans equal to the maximum allowed by the usury rate. The equilibrium value of the multiplier will be zero at scenarios in which the

[1] The USAID, in a September 2007 study, reported that 69% of the surveyed Colombian citizens have used, at least once, the informal banking sector as a source of credit. Among this population, those who knew the rate at which their loans were subjected to, reported an average of 64% monthly interest rate.

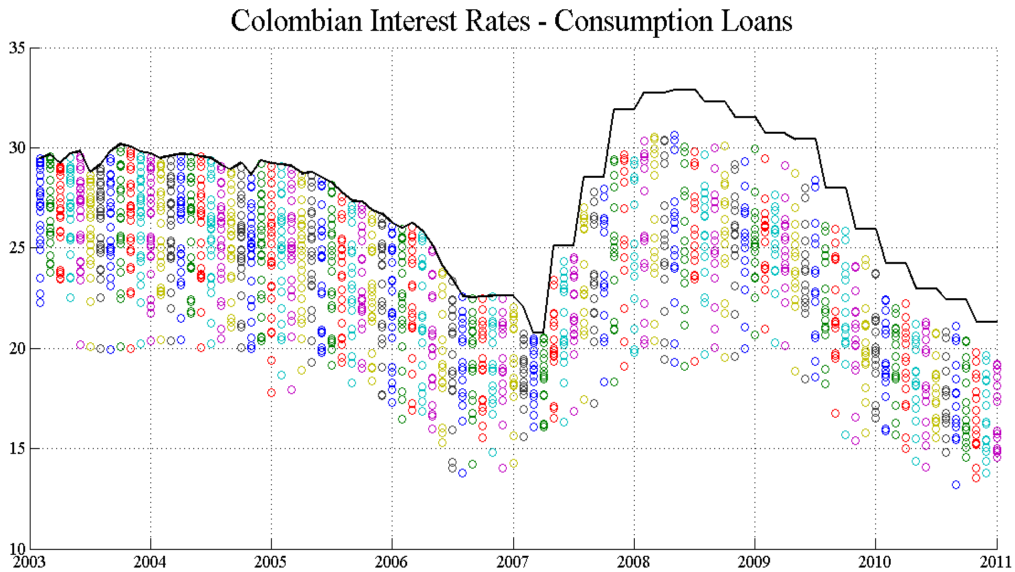

Figure 1: Average monthly rates of Colombian banking institutions for consumer loans, from 2003:01 to 2010:12. The solid black line indicates the "usury rate" or interest rate ceiling. Source of data: Superintendencia Superintendencia Financiera de Colombia. Diagram developed by author.

financial sector imposes an interest rate lower than the maximum allowed by the central government. The study of a DSGE model that includes a discriminating nonnegative multiplier associated with the interest rate ceiling enables a rigorous analysis on the asymmetries of credit cycles in economies subject to a usury rate. The latter is due to the fact that the restriction on credit operates only at excess levels of interest rate and not at lesser, causing an asymmetric cut in the optimal decision rules of a competitive equilibrium.

The perturbation method employed in Kiyotaki & Moore (1997) to approximate the equilibrium of its economic model is not a feasible methodology for approximating the equilibrium of the DSGE model proposed in this manuscript. That is because the solution technique in Kiyotaki & Moore (1997) requires the linearization of the non-linear system of equations representing the economic model around its steady state. When linearizing a system of equations, that is non-linear in nature, many dynamic elements of the model are lost to allow for a simple linear approximation; see Aruoba et al. (2006) and McGrattan (1996) for further reference. In addition, if a linearization of the system of equations were to be implemented, following the technique in Kiyotaki & Moore (1997), it would be necessary to impose the restriction that the interest rate financial institutions levy on debtors always equals the usury rate. An inspection of the average monthly interest rates on consumer loans offered by banking institutions in Colombia from 2003 to 2010, see Figure 1, reveals that these financial institutions do not always set their interest rates at the upper limit allowed by the usury rate. Therefore, to approximate the equilibrium of the economic system assuming that this restriction is the value which banking institutions always fix interest rates would be misleading and would disregard the complex dynamic nature that may rise in the model with respect to credit cycles. Using the data plotted in Figure 1 and the methodology advanced in Cho, et al. (2003) and Hsieh, et al. (2009), an analysis on the "magnet effect" of the usury rate over the interest rate of banks is to be developed. In the considered economic setting, the "magnet effect" is defined by an acceleration of interest rates to their upper bound even in scenarios where such price fixation is not optimal. The results of the economic analysis on the "magnet effect" will test the soundness of the predictions derived from the DSGE model.

An occasionally binding usury rate poses a challenge when approximating the solution of the model due to the resulting highly non-linear policy functions. For this reason, a projection method that does not require the linearization of the non-linear system of equations describing the first order conditions of the model economy and the limiting constraint on interest rates will be developed to approximate the policy functions that solve for the equilibrium of the economy. The proposed methodology is an adaptation of the Parameterized Expectations Algorithm (PEA), first introduced by Den Haan & Marcet (1994), where the system's true policy functions are parameterized using the Finite Element Method (FEM), in a manner similar to that proposed in McGrattan (1996). On a finite element approximation, the functional equation of the model is parameterized by subdividing the domain of the state space into nonintersecting subdomains called "elements", and fitting low-order polynomials on each subdomain. This procedure allows for accurate and stable approximations of the true policy functions in highly non-linear regions of the state space and, when combined with the parameterization of expectation functions, permits an efficient handling of occasionally binding constraints. Assessments on the speed and accuracy of finite elements in a parameterized expectation algorithm (FEM-PEA algorithm) are found in Cao-Alvira (2010) and Cao-Alvira (2011), where two algorithms with these properties are developed to approximate the solution of an optimal growth model with leisure subject to an irreversible investment constraint and a cash-in-advance constraint, respectively.

The next section contains a detailed description of the modeling environment and the functional forms of its equilibrium conditions. Section 3 discusses the proposed solution methodology and its implementation. Section 4 shows the global solution of the economy. Section 5 concludes.

2 Heterogeneous Agents Model

This section presents the planner's problem, its equilibrium conditions and the functional forms for which the solution procedure, discussed in Section 3, involving finite elements and a parameterized expectations algorithm, is implemented.

2.1 Discussion of the economy

The economic environment is modeled similar to Kiyotaki & Moore (1997) credit cycle general equilibrium model. In an economy with two types of agents i, gatherers and farmers $i = \{g, f\}$, a benevolent social planner maximizes the lifetime utility function of two infinitely lived representative agents of each type by making choices over consumption, labor supply, next period land, and bond holdings. Gatherers and farmers have different time discount factors, i.e. $\beta_g > \beta_f$. Farmers are subject to a credit constraint that limits the amount of claims they can issue. Both agents are subject to an anti-usury constraint, i.e. $R_t \leq \bar{R}_t$. It is sufficient that the constraint only enters in the gatherers objective function.

2.1.1 Gatherers

Given $h_{g,0}$ and $b_{g,0}$, the social planner chooses infinite sequences of consumption $\{c_{g,t}\}_{t=0}^{\infty}$, labor supply $\{n_{g,t}\}_{t=0}^{\infty}$, next period land $\{h_{g,t}\}_{t=0}^{\infty}$, and next period bond holdings $\{b_{g,t}\}_{t=0}^{\infty}$ for an infinitely lived agent to solve:

$$\max_{\{c_{g,t},n_{g,t},h_{g,t},b_{g,t}\}} E_t \left\{ \sum_{t=0}^{\infty} \beta_g^t \left\{ \frac{c_{g,t}^{1-\tau}}{1-\tau} - \gamma_g \frac{n_{g,t}^{1+\gamma}}{1+\gamma} \right\} \right\} \tag{1}$$

subject to the budget constraint:

$$c_{g,t} + q_t h_{g,t} + R_t b_{g,t-1} = Y_{g,t} + q_t h_{g,t-1} + b_{g,t} \tag{2}$$

and the anti-usury constraint:

$$R_t \leq \bar{R}_t \tag{3}$$

Production is assumed a CRS Cobb-Douglas with land and labor: $Y_{g,t} = A_{g,t} h_{g,t-1}^{\alpha_g} n_{g,t}^{1-\alpha_g}$, where $0 < \alpha_g < 1$. $\beta_g \in (0,1)$ is the time discount factor for the gatherers.

The production technology parameter for the gatherers evolves according to a first order autoregressive process, identified by the serial correlation parameter $\rho_g \in (-1,1)$, and normally distributed shocks ε_g :

$$\ln(A_{g,t}) = (1-\rho_g) A_g^{ss} + \rho_g \ln(A_{g,t-1}) + \varepsilon_{g,t} \quad \varepsilon_{g,t} \sim N\left(0, \sigma_g^2\right). \tag{4}$$

Defining $\lambda_{g,t}$ as the Lagrangian multiplier associated with the budget constraint and Π_t as the multiplier for the anti-usury constraint, the planner's problem is identified by the first order conditions in eqs. (5) – (8):

$$\lambda_{g,t} = c_{g,t}^{-\tau} + \frac{\Pi_t}{b_{g,t-1}} \tag{5}$$

$$\gamma_g n_{g,t}^{\gamma} = (1-\alpha_g) \left[\lambda_{g,t} - \frac{\Pi_t}{b_{g,t-1}} \right] \frac{Y_{g,t}}{n_{g,t}} \tag{6}$$

$$q_t \lambda_{g,t} - \frac{\Pi_t}{b_{g,t-1}} q_t = \beta_g E_t \left\{ \lambda_{g,t+1} \left(\alpha_g \frac{Y_{g,t+1}}{h_{g,t}} + q_{t+1} \right) - \frac{\Pi_{t+1}}{b_{g,t}} q_{t+1} \right\} \tag{7}$$

$$\lambda_{g,t} - \frac{\Pi_t}{b_{g,t-1}} = \beta_g E_t \left\{ \left(\lambda_{g,t+1} - \frac{\Pi_{t+1}}{b_{g,t}} \right) R_{t+1} \right\} \tag{8}$$

the Kuhn-Tucker condition in eq. (9) :

$$R_t \leq \bar{R}_t \quad \text{and} \quad \Pi_t \left[\bar{R} - R_t \right] = 0 \tag{9}$$

and the market clearing conditions for the goods market, in eq. (2).

2.1.2 Farmers

Given $h_{f,0}$ and $b_{f,0}$, the social planner chooses infinite sequences of consumption $\{c_{f,t}\}_{t=0}^{\infty}$, labor supply $\{n_{f,t}\}_{t=0}^{\infty}$, next period land $\{h_{f,t}\}_{t=0}^{\infty}$, and next period bond holdings $\{b_{f,t}\}_{t=0}^{\infty}$ for an infinitely lived agent to solve:

$$\max_{\{c_{f,t}, n_{f,t}, h_{f,t}, b_{f,t}\}} E_t \left\{ \sum_{t=0}^{\infty} \beta_f^t \left\{ \frac{c_{f,t}^{1-\tau}}{1-\tau} - \gamma_g \frac{n_{f,t}^{1+\gamma}}{1+\gamma} \right\} \right\} \tag{10}$$

subject to the budget constraint:

$$c_{f,t} + q_t h_{f,t} + R_t b_{f,t-1} = Y_{f,t} + q_t h_{f,t-1} + b_{f,t} \tag{11}$$

and the credit constraint:

$$E_t \{R_{t+1}\} b_{f,t} \leq \beta_f M E_t \{q_{t+1}\} h_{f,t} \tag{12}$$

Production is assumed a CRS Cobb-Douglas with land and labor: $Y_{f,t} = A_{f,t} h_{f,t-1}^{\alpha_g} n_{f,t}^{1-\alpha_g}$, where $0 < \alpha_f < 1$. $\beta_f \in (0,1)$ is the time discount factor for the gatherers.

The production technology parameter for the farmers evolves according to a first order autoregressive process, identified by the serial correlation parameter $\rho_f \in (-1,1)$, and normally distributed shocks ε_f :

$$\ln(A_{f,t}) = (1-\rho_f) A_f^{ss} + \rho_f \ln(A_{f,t-1}) + \varepsilon_{f,t} \quad \varepsilon_{f,t} \sim N\left(0, \sigma_f^2\right). \tag{13}$$

Defining $\lambda_{f,t}$ as the Lagrangian multiplier associated with the budget constraint and $\mu_{f,t}$ as the multiplier for the credit constraint, the planner's problem is identified by the first order conditions in eqs. (14) – (17):

$$c_{f,t}^{-\tau} = \lambda_t \tag{14}$$

$$\gamma_f n_{f,t}^{\gamma} = \lambda_{f,t} (1-\alpha_g) \frac{Y_{f,t}}{n_{f,t}} \tag{15}$$

$$q_t \lambda_{f,t} = \beta_f E_t \left\{ \lambda_{f,t+1} \left(\alpha_f \frac{Y_{f,t+1}}{h_{f,t}} + (1 + M\mu_t) q_{t+1} \right) \right\} \tag{16}$$

$$\lambda_{f,t} = \beta_f E_t \{ (\lambda_{f,t+1} + \mu_t) R_{t+1} \} \tag{17}$$

the Kuhn-Tucker condition in eq. (18):

$$E_t \{R_{t+1}\} b_{f,t} \leq \beta_f M E_t \{q_{t+1}\} h_{f,t} \quad \text{and} \quad \mu_t \left[\beta_f M E_t \{q_{t+1}\} h_{f,t} - E_t \{R_{t+1}\} b_{f,t} \right] = 0 \tag{18}$$

and the market clearing conditions for the goods market, in eq. (2).

2.1.3 Market clearing conditions

The asset supply is normalize to one, and bonds emited by an agent are the debt of the other:

$$h_{g,t} + h_{f,t} = 1 \tag{19}$$

$$b_{g,t} + b_{f,t} = 0 \tag{20}$$

2.1.4 Exogenous shock to the economy

The state vector $\Lambda = \left[\bar{A_g};\bar{A_f}\right]$ of exogenous technology parameters evolves according to a Markovian process with transitional probabilities Π. Q is the number of possible states of nature of Λ, $\sum_{z=1}^{Q} \Pi_{wz} = 1$. For each $w = \{1, ..., Q\}$ & $z = \{1, ..., Q\}$, typical element Π_{wz} is the probability of being on state z on time $t+1$ given the realization of state w in time t:

$$\Pi_{wz} = \Pr\left[\Lambda_{t+1} = \Lambda(z) | \Lambda_t = \Lambda(w)\right]. \tag{21}$$

2.2 Equilibrium of the economy

The equilibrium for the heterogeneous agents economy is denoted by the sequence of variables $\{\digamma_t\}_{t=0}^{\infty} = \{c_{g,t}, c_{f,t}, n_{g,t}, n_{f,t}, h_{g,t}, h_{f,t}, b_{g,t}, b_{f,t}, R_t, q_t\}_{t=0}^{\infty}$, given a sequence of exogenous parameters $\{A_{g,t}, A_{f,t}\}_{t=0}^{\infty}$ evolving according to the transition matrix Π_{wz} in (21), and initials stock of land h_0 and bond holdings b_0, which satisfy the first order conditions (5), (14), (6), (15), (7), (16), (8) & (17), the credit constraint (18), the anti-usury constraint (9), the flow of funds conditions (2) & (11), and the market clearing conditions (19) & (20).

2.3 State space & functional forms of the economy

Θ is the state space of the economy, which can be sub-divided in two subsets; one, Ω, containing the continuous variables of the state space, and a second, Λ, containing the discrete state variables. At time t, the partial state space Ω_t is composed of the possible realizations of land at time $t-1$ and the bond holdings at time $t-1$. Ω has a well defined compact support, i.e. $\Omega = \left[\underline{\mathrm{h}}, \bar{\mathrm{h}}\right] \times \left[\underline{\mathrm{b}}, \bar{\mathrm{b}}\right]$. Λ_t is composed of the possible realizations of the technology parameters at time t. Λ also has a well defined compact support, i.e. $\Lambda = [A(1), ..., A(Q)]$.

The solution methodology employs the use of time invariant policy functions B_w^f, H_w^f, Q_w, N_w^f, N_w^g,Φ_w& λ_w^g for $w = [1, ..., Q]$, to express the equilibrium conditions of the credit model economy. Conditional on $A_t = A(w)$, $B_w^f(\Omega_t)$ and $H_w^f(\Omega_t)$ are respectively defined to map the previous state of farmer's land and bond holdings into their next periods land and bond holdings. Q_w the previous state space into current period's land prices. $N_w^f(\Omega_t)$ and $N_w^g(\Omega_t)$ respcetively map the current state space into the control of the labor supply function of farmers and gatherers. Φ_w maps the current state space into a portion of the conditional expectation function of the farmer's current period's optimal bond holdings i.e. i.e. $\Phi_w(\Omega_t) \equiv \Phi_{\Theta_t} = \Phi_w\left(h_{t-1}, b_{t-1}|\theta_t = \theta(w)\right)$ where $\Phi_{\Theta_t} = \beta_f E_t\left\{\lambda_{f,t+1}R_{t+1}\right\}$. $\lambda^g_w(\Omega_t)$ maps the current state space into the value of the gatherer's budget constraint multiplier.

Define Γ as a matrix containg the column vectors $\bar{B}^f$, $\bar{H}^f$, $\bar{Q}$, $\bar{N}^f$, $\bar{N}^g$,$\bar{\Phi}$ and $\bar{\lambda}$ of the previously described policy functions, and $\Upsilon_w = \lambda^g_w - \frac{\Pi_w}{-b}$. Using the functional forms of the policy functions and the Markovian nature of the exogenous parameters, the residuals of the Euler equation and the equilibrium condition for real money holdings can be written as in eqs (18) and (18).

$$R^{ng}_w\left(h, b; \Gamma\right) = \gamma_g N^g_w\left(\Omega_t\right)^{\gamma} - \left(1-\alpha_g\right)\Upsilon_w \frac{Y_{g,w}}{N^g_w\left(\Omega_t\right)} \tag{22}$$

$$R^{nf}_w\left(h, b; \Gamma\right) = \gamma_f N^f_w\left(\Omega_t\right)^{\gamma} - \left(1-\alpha_f\right)\lambda_{f,w} \frac{Y_{f,w}}{N^f_w\left(\Omega_t\right)} \tag{23}$$

$$R^{hg}_w\left(h, b; \Gamma\right) = \Upsilon_w Q_w\left(\Omega_t\right) - \beta_g E_t\left\{\lambda^g_w(\Omega_t)\left(\alpha_g \frac{\tilde{Y}_{g,z}}{\left(1-H^f_w(\Omega_t)\right)}\right) + \tilde{\Upsilon}_z Q_z\left(\Omega_{t+1}\right)\right\} \tag{24}$$

$$R^{bg}_w\left(h, b; \Gamma\right) = \Upsilon_w\left(\Omega_t\right) - \beta_g E_t\left\{\Upsilon_z\left(\Omega_{t+1}\right)\tilde{R}_z\right\} \tag{25}$$

$$R^{hf}_w\left(h, b; \Gamma\right) = \lambda^f_w Q_w\left(\Omega_t\right) - \beta_f E_t\left\{\tilde{\lambda}_f\left(\alpha_f \frac{\tilde{Y}_{f,z}}{H^f_w(\Omega_t)} + \left(1+M\mu_w\right)Q_z\left(\Omega_{t+1}\right)\right)\right\} \tag{26}$$

$$R^{bf}_w\left(h, b; \Gamma\right) = \Upsilon_w\left(\Omega_t\right) + \mu_w\beta_f E_t\left\{\tilde{R}_z\right\} - \beta_f E_t\left\{\tilde{\lambda}^f_z\tilde{R}_z\right\} \tag{27}$$

$$R^{ccf}_w\left(h, b; \Gamma\right) = \mu_w\left[\beta_f M E_t\left\{Q_z\left(\Omega_{t+1}\right)\right\}H^f_z(\Omega_{t+1}) - E_t\left\{\tilde{R}_z\right\}B_z\left(\Omega_{t+1}\right)\right] \tag{28}$$

for all $w = \{1, ..., Q\}$. The real variables are defined by

$$Y_{g,w} = A_g(w)\left(1-h\right)^{\alpha_g} N^g_w\left(\Omega_t\right)^{1-\alpha_g} \tag{29}$$

$$\Pi_w = b\left({c_{g,w}}^{-\tau} - \lambda_w(\Omega_t)\right) \tag{30}$$

$$c_{g,w} = \begin{cases} \lambda_w(\Omega_t)^{-\frac{1}{\tau}} & \text{if} \quad \Pi_w = 0 \\ Y_{g,w} + Q_w\left(\Omega_t\right)\left(1-h\right) - B_w\left(\Omega_t\right) - Q_w\left(\Omega_t\right)\left(1-H_w(\Omega_t)\right) + R_w b & \text{if} \quad \Pi_w > 0 \end{cases} \tag{31}$$

$$R_w = \begin{cases} \left[Y_{g,w} + Q_w\left(\Omega_t\right)\left(1-h\right) - B_w\left(\Omega_t\right) - c_{g,w} - Q_w\left(\Omega_t\right)\left(1-H_w(\Omega_t)\right)\right]/(-b) & \text{if} \quad \Pi_w = 0 \\ \bar{R}_t & \text{if} \quad \Pi_w > 0 \end{cases} \tag{32}$$

$$Y_{f,w} = A_f(w)h^{\alpha_f}N^f_w\left(\Omega_t\right)^{1-\alpha_f} \tag{33}$$

$$c_{f,w} = Y_{f,w} + Y_{g,w} - c_{g,w} \tag{34}$$

$$\lambda_{f,w} = c^{-\tau}_{f,w} \tag{35}$$

$$\mu_w = \frac{\lambda_{f,w} - \Phi_w(\Omega_t)}{\beta_f E_t \left\{ \tilde{R}_z \right\}} \tag{36}$$

Policy functions $\bar{B}^f$, $\bar{H}^f$, $\bar{Q}$, $\bar{N}^f$, $\bar{N}^g$,$\bar{\Phi}$ and $\bar{\lambda}$ are the equilibrium solutions to $R_w^{ng}(h, b; \Gamma) = 0$, $R_w^{nf}(h, b; \Gamma) = 0$, $R_w^{hg}(h, b; \Gamma) = 0$, $R_w^{bg}(h, b; \Gamma) = 0$, $R_w^{hf}(h, b; \Gamma) = 0$, $R_w^{bf}(h, b; \Gamma) = 0$ and $R_w^{ccf}(h, b; \Gamma) = 0$ for all w, described in eqs. (22)-(28), and in combination with $Y_{g,w}$, $Y_{f,w}$, $c_{g,w}$, $c_{f,w}$, Π_w, R_w, μ_w and $\lambda_{f,w}$ from eqs. (29)-(30), and the sequence of techonology parameters in Λ, evolving according the transition matrix Π in (21), generate the sequences $\{c_{g,t}, c_{f,t}, n_{g,t}, n_{f,t}, h_{g,t}, h_{f,t}, b_{g,t}, b_{f,t}, R_t, q_t\}_{t=0}^{\infty}$ that solve for the equilibrium of this economy along the state space Θ.

3 Solution Methodology

The solution procedure uses finite elements in approximating the policy functions and a parameterized expectations algorithm to minimize the weighted absolute value of residual functions $R_w^{ng}(h, b; \Gamma)$, $R_w^{nf}(h, b; \Gamma)$, $R_w^{hg}(h, b; \Gamma)$, $R_w^{bg}(h, b; \Gamma)$, $R_w^{hf}(h, b; \Gamma)$, $R_w^{bf}(h, b; \Gamma)$ and $R_w^{ccf}(h, b; \Gamma)$for all w; where the true decision rules B_w^f, H_w^f, Q_w, N_w^f, N_w^g,Φ_w& λ_w^g are replaced by the parametric approximations $b_w^h(\Omega_t)$, $h_w^h(\Omega_t)$, $q_w^h(\Omega_t)$, $n_{f,w}^h(\Omega_t)$, $n_{g,w}^h(\Omega_t)$, $\phi_w^h(\Omega_t)$ and $\upsilon_w^h(\Omega_t)$. b_w^h, h_w^h, q_w^h, $n_{f,w}^h$, $n_{g,w}^h$, ϕ_w^h and υ_w^h. are approximated using an implementation of the finite element method, that follows that advocated in McGrattan (1996).

To create the approximate time invariant functions b_w^h, h_w^h, q_w^h, $n_{f,w}^h$, $n_{g,w}^h$, ϕ_w^h and υ_w^h, the space $\Omega = \left[\underline{\mathrm{k}}, \bar{\mathrm{k}}\right] \times \left[\underline{\mathrm{M}}, \bar{\mathrm{M}}\right]$ is divided in n_e nonoverlapping rectangular subdomains called "elements". At each realization of w, the parameterizations of the policy functions for each element are constructed using linear combinations of low order polynomials or "basis functions". This procedure creates local approximations for each function. Given the discrete nature of Λ, this state space need not to be divided.

The parameterized functions b_w^h, h_w^h, q_w^h, $n_{f,w}^h$, $n_{g,w}^h$, ϕ_w^h and υ_w^h are built as follows:

$$b_w^h(h, b) = \sum_{ij}^{IJ} b_{ij}^w W_{ij}(h, b) \tag{37}$$

$$h_w^h(h, b) = \sum_{ij}^{IJ} h_{ij}^w W_{ij}(h, b) \tag{38}$$

$$q_w^h(h,b)=\sum_{ij}^{IJ} q_{ij}^w W_{ij}(h,b) \tag{39}$$

$$n_{f,w}^h(h,b)=\sum_{ij}^{IJ} n_{f,ij}^w W_{ij}(h,b) \tag{40}$$

$$n_{g,w}^h(h,b)=\sum_{ij}^{IJ} n_{g,ij}^w W_{ij}(h,b) \tag{41}$$

$$\phi_w^h(h,b)=\sum_{ij}^{IJ} \phi_{ij}^w W_{ij}(h,b) \tag{42}$$

$$\upsilon_w^h(k,M)=\sum_{ij}^{IJ} \upsilon_{ij}^w W_{ij}(h,b). \tag{43}$$

Where $i=\{1,...,I\}$ indicate capital nodes, $j=\{1,...,J\}$ indicate money supply nodes, $W_{ij}(h,b)$ is a set of linear basis functions dependent on the element $[h_i,h_{i+1}]\times[b_j,b_{j+1}]$, for all i,j, over which the local approximations are performed. b_{ij}^w, h_{ij}^w, q_{ij}^w, $n_{f,ij}^w$, $n_{g,ij}^w$, ϕ_{ij}^w, and υ_{ij}^w are vectors of coefficients to be determined. The parameterized value of the conditional expectation function and the price function over the full state space are obtained by piecing together all the local approximations. The approximate solutions for $b_w^h(\Omega_t)$, $h_w^h(\Omega_t)$, $q_w^h(\Omega_t)$, $n_{f,w}^h(\Omega_t)$, $n_{g,w}^h(\Omega_t)$, $\phi_w^h(\Omega_t)$ and $\upsilon_w^h(\Omega_t)$ are then "piecewise linear functions" on Θ.

$W_{ij}(h,b)$ are the basis functions that the finite element method employs. These are constructed such that they take a value of zero for most of the space Ω, except for a small interval where they take a simple linear form. The basis functions adopted for these approximations are set such that $W_{ij}(h,b)=\Psi_i(h)\Sigma_j(b)$, where

$$\Psi_i(h)=\begin{cases}\frac{h-h_{i-1}}{h_i-h_{i-1}} & \text{if } h\in[h_{i-1},h] \\ \frac{h_{i+1}-h}{h_{i+1}-h_i} & \text{if } h\in[h_i,h_{i+1}] \\ 0 & \text{elsewhere}\end{cases} \tag{44}$$

$$\Sigma_j(b)=\begin{cases}\frac{b-b_{j-1}}{b_j-b_{j-1}} & \text{if } b\in[b_{j-1},b_j] \\ \frac{b_{j+1}-b}{b_{j+1}-b_j} & \text{if } b\in[b_j,b_{j+1}] \\ 0 & \text{elsewhere}\end{cases} \tag{45}$$

for all $i,\ j$. $\Psi_i(h)$ & $\Sigma_j(b)$ have the shape of a continuous pyramid which peaks at nodal points $h=h_i$ & $b=b_j$, respectively, and are non-zero only on the surrounding elements of these nodes. $R_w^{ng}(h,b;\Gamma)$, $R_w^{nf}(h,b;\Gamma)$, $R_w^{hg}(h,b;\Gamma)$, $R_w^{bg}(h,b;\Gamma)$, $R_w^{hf}(h,b;\Gamma)$, $R_w^{bf}(h,b;\Gamma)$ and $R_w^{ccf}(h,b;\Gamma)$

The approximations $b_w^h(\Omega_t)$, $h_w^h(\Omega_t)$, $q_w^h(\Omega_t)$, $n_{f,w}^h(\Omega_t)$, $n_{g,w}^h(\Omega_t)$, $\phi_w^h(\Omega_t)$ and $\upsilon_w^h(\Omega_t)$ are chosen to simultaneously satisfy the seven equations:

$$\int_{\underline{\mathrm{b}}}^{\bar{\mathrm{b}}}\int_{\underline{\mathrm{h}}}^{\bar{\mathrm{h}}}\omega\left(h,b\right)R_w^S\left(h,b;\Gamma\right)dhdb=0 \tag{46}$$

for $l=\{ng,\,nf,\,hg,\,bg,\,hf,\,bf,\,ccf\}$ and all $w=\{1,...,Q\}$. $\omega\left(h,b\right)$ is a weighting function, and $R_w^S(h,b;\Gamma)$ for all S are the residual functions in eqs (18) and (18) where the true policy functions in Γ are replaced by the vectors of parametric approximations $b_w^h(\Omega_t)$, $h_w^h(\Omega_t)$, $q_w^h(\Omega_t)$, $n_{f,w}^h(\Omega_t)$, $n_{g,w}^h(\Omega_t)$, $\phi_w^h(\Omega_t)$ and $\upsilon_w^h(\Omega_t)$. A Galerkin scheme is employed to find the vectors of coefficients b_{ij}^w, h_{ij}^w, q_{ij}^w, $n_{f,ij}^w$, $n_{g,ij}^w$, ϕ_{ij}^w, and υ_{ij}^w for all i,j and all w, that solve for the weighted residual functions in equations (46) over the complete space Θ. A Galerkin scheme uses the basis functions $W_{ij}\left(h,b\right)$ as weights for $R_w^S\left(h,b;\Gamma\right)$ for all l:

$$\int_{\underline{\mathrm{b}}}^{\bar{\mathrm{b}}}\int_{\underline{\mathrm{h}}}^{\bar{\mathrm{h}}}W_{ij}\left(h,b\right)R_w^S\left(h,b;\Gamma\right)dhdb=0 \tag{47}$$

for all l and all i, j and all states w. Since the basis functions are only nonzero surrounding their nodes, eqs. (47) can be rewritten in terms of the individual elements:

$$\sum_{e=1}^{n_e}\int_{\Omega_e}W_{ij}\left(k,M\right)R_w^K\left(h,b;\bar{\upsilon}^h,\bar{p}^h\right)dkdM=0 \tag{48}$$

for all l and all states w. n_e is the total number of elements and Ω_e is the land and bond holdings domain covered by the element e. Gauss-Legendre abscissas are used for the integration along the domain of h and b on each element.

A Newton algorithm is used to find the coefficients for $\varphi_s=\left[\bar{b}_s,\bar{h}_s,\bar{q}_s,\bar{n}_{f,s},\bar{n}_{g,s},\bar{\phi}_s,\bar{\upsilon}_s\right]$ which solve for the nonlinear system of equations H:

$$H\left(\varphi_s\right)=0. \tag{49}$$

Where $H\left(\varphi_s\right)$ is denoted by eq. (48) for $l=\{ng,\,nf,\,hg,\,bg,\,hf,\,bf,\,ccf\}$. The first step is to choose initial vectors of coefficients φ_{s_0}, and iterate as follows:

$$\varphi_{s_{l+1}}=\varphi_{s_l}-J\left(\varphi_{s_l}\right)^{-1}H\left(\varphi_{s_l}\right). \tag{50}$$

J is the Jacobian matrix of H, and φ_{s_l} is the l^{th} iteration of φ_s. Convergence is assumed to have occurred once $\left\|\varphi_{s_{l+1}}-\varphi_{s_l}\right\|<10^{-7}$. The algorithm solves for the parametric approximations $b_w^h(\Omega_t)$, $h_w^h(\Omega_t)$, $q_w^h(\Omega_t)$, $n_{f,w}^h(\Omega_t)$, $n_{g,w}^h(\Omega_t)$, $\phi_w^h(\Omega_t)$ and $\upsilon_w^h(\Omega_t)$ for $\{A\left(w\right)\}\subset\Lambda$ and $\{h,b\}\subset\Omega$.

Table 1: Calibration parameters of the benchmark economy

A_g^{ss}	A_f^{ss}	α_g	α_f	β_f	β_g	τ	γ	M	ρ_g	ρ_f	σ_g^2	σ_f^2	$\bar{R}_t$
0.86	0.90	0.33	0.33	0.97	0.99	1.0	0.0	0.50	1.0	0.9	0.0	0.03	∞

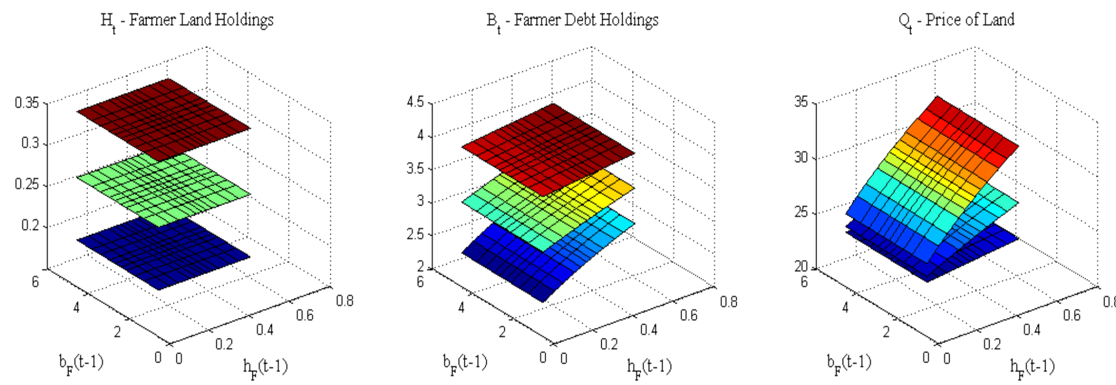

Figure 2: The three panels illustrate the parameterized versions of equations B_w^f (left), H_w^f (middle) and Q_w (right).

4 Business Cycle Properties

This section presents the equilibrium values assumed by the endogenous variables of the model resulting from the global approximation method. Additionally, a simulation exercise is performed where the model is shocked exogenously by a Markovian money growth rate series.

4.1 Calibration

The model calibration is typical to that of the literature. The time interval is a quarter of a year. Gatherers are assumed discount less the future than farmers and being less productive; i.e. $\beta_g > \beta_f$ and $A_{g,t} < A_{f,t}$. The time discount factor for gatherers is set to 0.99 and for farmers to 0.97, and the productivity parameters are 0.86 and 0.90 respectively. The capital elasticity of output to 1/3. The intertemporal elasticity of substitution in consumption is set to equal 1, e.g. $\tau = 1.00$, equivalent a logarithmic utility. The inverse of labor supply elasticity is set to 0, denoting Hansen (1985) indivisible labor. The calibration parameters are summarized in Table (1).

4.2 State space partition and Global Solution to the economy

The state space corresponding to the previous period's land holdings is partitioned on seven points points, i.e. $I = 7$: $h = [0.05; 0.15; 0.25; 0.30; 0.35; 0.45; 0.55]$. The state space corresponding to the pre-

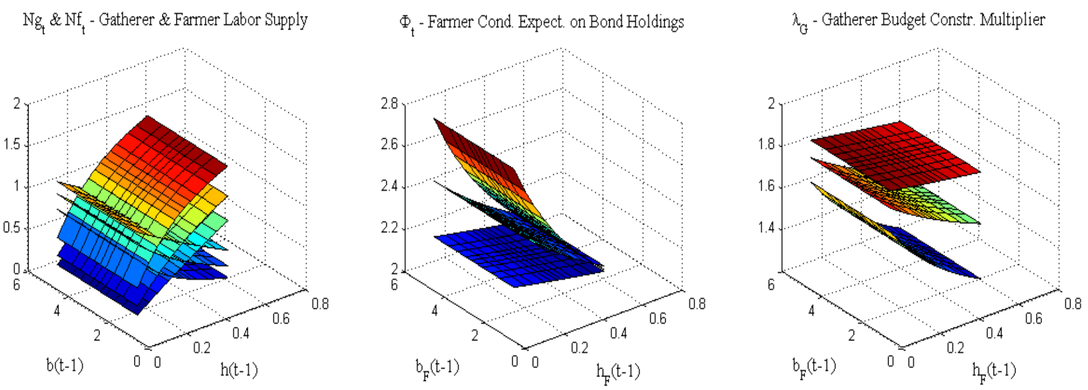

Figure 3: The three panels illustrate N_w^f and N_w^g (left),Φ_w (middle) and λ_w^g (right).

vious period's bond holdings is partitioned on six points, i.e. $J = 6$: $b = [1.00; 2.25; 3.0; 3.50; 4.25; 5.50]$. The AR(1) process of the technology parameter is approximated by a discrete Markov chain using the methodology advanced by Tauchen and Hussey (1991). Given the high persistence of the autoregressive process, an adjustment to the weighting function is performed, as recommended by Flodén (2008). Three states for the farmer's technology parameter are considered, $Q = 3$; these return the state vector $\Lambda = [0.83,\ 0.90,\ 0.97]$. The total amount of elements on the global approximation is 90 $(= (I-1) \times (J-1) \times Q)$.

The panels of Figure 2 and Figure 3 illustrate the parameterized versions of the conditional expectation function of B_w^f, H_w^f, Q_w, N_w^f, N_w^g,Φ_w& λ_w^g for a global approximate solution of the economy. Two Gauss-Legendre abscissas are used on the h and b domains of each element. The approximation of the stationary competitive equilibrium lies within the support of the state space.

4.3 Effects of an Anti-Usury Constraint

Figures (4) and (5) depict the equilibrium values of selected variables for two determinate versions of the credit model. One is the benchmark scenario, where the usury rate is sufficiently high that the anti-usury constraint does not bind. The alternate version of the model considers a usury rate of $\bar{R}_t = 2.0175$, where the constraint binds at values greater than $h_{f,t-1} \geq 0.34$. The left panel of Figure (3) depicts the values of the slackness multiplier of the anti-usury rate Π_t on both scenarios, the middle panel depicts the price of land q_t, and the right panel depicts the farmer's land holdings $h_t$for

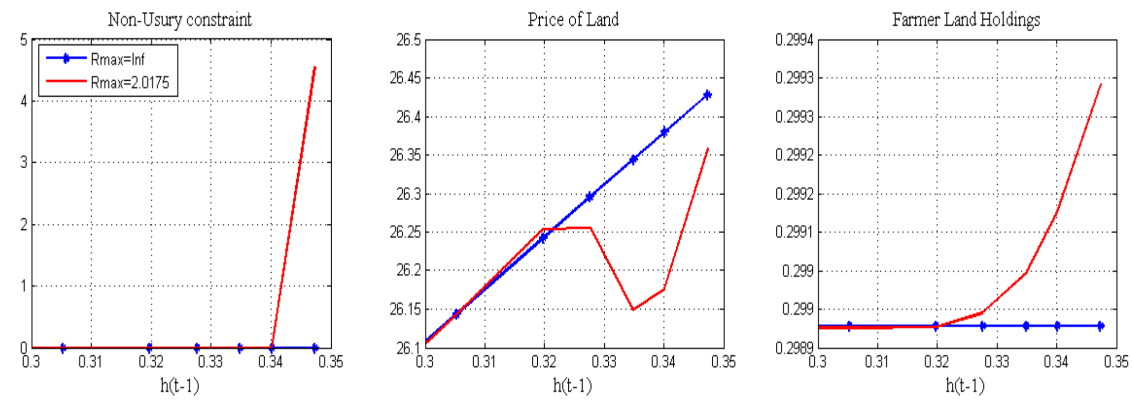

Figure 4: The left panel depicts the values of the slackness multiplier of the anti-usury rate Π_t on both scenarios, the middle panel depicts the price of land q_t, and the right panel depicts the farmer's land holdings $h_t$for next period.

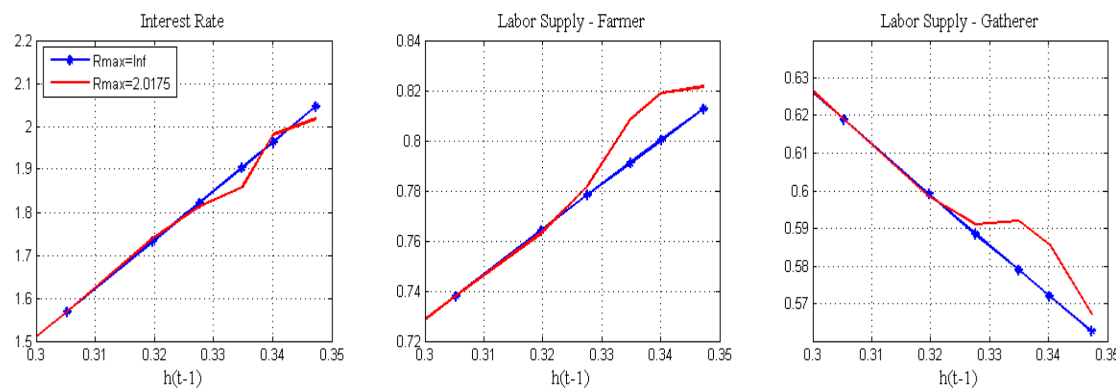

Figure 5: The left panel depicts the values of the interest rate R_t on both scenarios, the middle panel depicts the labor supply of the farmer $n_{f,t}$, and the right panel depicts the labor supply of the gatherer $n_{g,t}$.

next period. Bindingness of the anti-usury constraint decreases the price of land in the economy and increases the land holdings of the productive agent. The left panel of Figure (4) depicts the values of the interest rate R_t on both scenarios, the middle panel depicts the labor supply of the farmer $n_{f,t}$, and the right panel depicts the labor supply of the gatherer $n_{g,t}$. The imposition of a binding anti-usury constraint has the effect of increasing the labor supply of both types of agents.

5 Conclusion

This paper solves for a heterogenous agent dynamic stochastic general equilibrium model affected by two types of credit constraints, a constraint on the total amount of credit available to productive agents and constrained by a ceiling in the interest rate, or a usury rate. The model is solved using an approximation technique involving finite elements and a parameterized expectations algorithm. The solution method efficiently allows for the constraints to occasionally bind, permitting an analysis on the nonlinear analysis on the credit market. By including a discriminating nonnegative multiplier associated with the interest rate ceiling on the DSGE model, a rigorous analysis is enabled on the asymmetries of credit cycles in economies subject to a usury rate. The latter is due to the fact that the restriction on credit operates only at excess levels of interest rate and not at lesser, causing an asymmetric cut in the optimal decision rules of a competitive equilibrium. The impact of both credit constraints on the real variables of the model economy is assessed.

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